4. DEFLECTION ON FALLING (8 points) — *Mihkel Kree.* Imagine a vertical mine shaft of height $h = 100\,\mathrm{m}$ at the equator. Consider a free falling steel ball, released from the top of the shaft, and let us neglect the air drag (friction). Not surprisingly, due to the rotation of the Earth, the ball will reach the bottom of the shaft at a point which is slightly different from the point of the vertical projection of the release location. Let us denote the distance between these points by Δx. (Historical note: the correct expression for Δx was first calculated independently by Laplace and Gauss in 1803.)

i) *(1 point)* Calculate the velocity difference Δv of the top of the shaft and the bottom of the shaft as seen in an absolute, non-rotating frame of reference.

ii) *(1 point)* Neglecting the rotation of the Earth, but assuming that the steel ball is released with an initial horizontal velocity Δv as seen from the bottom of the shaft, calculate the horizontal displacement Δx_0 of the steel ball by the time it reaches the bottom.

Obviously, the answer found in ii) is not physically correct, because we neglected the rotation of the Earth and, correspondingly, the Coriolis force. Luckily, the correct expression can still be found without integrating the Coriolis force (although Laplace and Gauss did it).

iii) *(6 points)* Considering the falling steel ball being on a Keplerian orbit and taking into account the curvature of the Earth, find the true value of the horizontal displacement Δx. *Hint: use appropriate approximations for small values and note that the small segment of ellipse corresponding to the fall of the steel ball can still be approximated as a parabola.*