4. GRAVITATIONAL WAVES (7 points) — *Artūrs Bērziņš.* The power radiated in gravitational waves by an orbiting binary system is given by

$$P(r, m_1, m_2) = \frac{32}{5} \frac{G^4}{c^5} \frac{(m_1 m_2)^2 (m_1 + m_2)}{r^5},$$

where r is the distance between the centers of the two orbiting masses m_1 and m_2. It is known that the most compact object is a black hole. The size of a black hole is defined by its Schwarzschild radius $r_s = \frac{2Gm}{c^2}$, where m is the mass of the black hole.

i) *(2 points)* Estimate the upper limit of the power that can ever be emitted in gravitational waves by an orbiting binary system.

The gravitational wave detectors on Earth function by measuring the so called gravity wave strain $\varepsilon(t)$ over time, which characterizes the deformation of spacetime. Data processing then yields the maximum strain ε and its corresponding wave frequency f. With the help of a theoretical spacetime model, the energy density u associated with the wave can then be determined. We will use the analogy of linear elasticity to examine this model.

ii) *(1.5 points)* Derive the energy density $u = u(\varepsilon, E)$ in a uniformly stretched elastic band in terms of the strain ε and the Elastic (Young's) modulus E.

iii) *(1.5 points)* Use dimensional analysis to estimate the frequency-dependent elastic modulus of spacetime $E(f)$ in terms of the universal gravitational constant G, speed of light c and gravitational wave frequency f.

iv) *(2 points)* Estimate the maximum distance $z = z(\varepsilon, f)$ from the Earth to the gravitational wave source as a function of the strain ε and frequency f. Use the models derived previously in this problem.