7. ZENER

i) The diode limits the maximum amount of voltage that can be applied to the inductor. Whenever the current from the current source is higher than the current through the inductor, the voltage V_1 is applied to the inductor, and when the current current source is lower than the current through the inductor a voltage $-V_2$ is applied to the inductor. If the largest voltage V_1 would be applied to the inductor for one period $T = \frac{2\pi}{\omega}$, then the change in the current through the inductor is

$$\delta I = \frac{V_1}{L} T = \frac{2\pi V_1}{L\omega}.$$

Because we had the relation $L\omega I_0 \gg V_1$, we get that $\delta I \ll I_0$. The current through the inductor changes negligibly little compared to I_0 during one period and can be considered constant. The equilibrium current can be found by finding when the average voltage is zero. During one period let T_1 be the time when $\langle I \rangle > I_0 \cos \omega t$, and let T_2 be the time when $\langle I \rangle < I_0 \cos \omega t$. We get

$$T_1 = \frac{2}{\omega} \arccos\left(\frac{\langle I \rangle}{I_0}\right).$$

The average voltage is zero when $V_1 T_1 = V_2 T_2$ and by using $T_1 + T_2 = T$ we have

$$V_1 T_1 = V_2(T - T_1) \Longrightarrow T_1 = \frac{V_2}{V_1 + V_2} T \Longrightarrow$$

$$\frac{2}{\omega} \arccos\left(\frac{\langle I \rangle}{I_0}\right) = \frac{V_2}{V_1 + V_2} \frac{2\pi}{\omega} \Longrightarrow$$

$$\langle I \rangle = I_0 \cos\left(\frac{\pi V_2}{V_1 + V_2}\right).$$

ii) The peak-to-peak amplitude of the current fluctuations can be calculated from $\frac{dI}{dt} = \frac{U}{L}$.

$$\Delta I = \frac{V_2 T_2}{L} = \frac{V_1 T_1}{L} = \frac{2\pi V_1 V_2}{\omega L (V_1 + V_2)}.$$