9. SPACECRAFT PRESSURE

i) Consider an air slice between r and $r + dr$ with area A and mass m. In a rotating frame of reference, force balance gives

$$A(p(r + dr) - p(r)) = m\omega^2 r.$$

The mass is related to the density with $m = A\rho(r)dr$. Using the ideal gas law,

$$pV = nRT \Longrightarrow$$

$$\rho(r) = \frac{m}{V} = \frac{m}{nRT}p(r) = \frac{\mu}{RT}p(r),$$

so that when we combine the two equations, we get the following equation for pressure:

$$\frac{p(r + dr) - p(r)}{dr} = p'(r) = \frac{\mu\omega^2}{RT} r\, p(r).$$

This can be solved using separation of variables, giving

$$\int \frac{d\rho}{\rho} = \int \frac{\mu\omega^2}{RT} r\, dr,$$

$$\ln \rho - \ln \rho_0 = \frac{\mu\omega^2}{2RT} r^2.$$

From this we find

$$p(r) = p_0 \exp\left(\frac{\mu\omega^2}{2RT} r^2\right).$$

ii) This time the force balance is

$$A(p(h + dh) - p(h)) = -mg$$

The corresponding pressure equation is

$$p'(h) = -\frac{\mu g}{RT} p(h)$$

with the solution

$$p(h) = p_0 \exp\left(-\frac{\mu g}{RT} h\right).$$