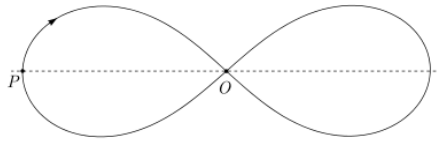
1. Gravitational racing (11 points) — *Maté Vigh and Jaan Kalda.*

While in generic case, the dynamics of three gravitationally interacting bodies is complicated and chaotic, there are special cases when the dynamics is regular. In particular, there are cases when bodies perform periodic motion. The simplest case of such periodic motion is when all three bodies, positioned at the vertices of an equilateral triangle, rotate as a rigid body. In what follows we consider a more complicated periodic motion.

Relatively recently[1], it was discovered that three equal point masses can move periodically along a common 8-shaped trajectory shown in the figure (arrow denotes the direction of motion). This figure is based on a computer simulation and has a correct shape. If needed, you can measure distances from the enlarged version of the figure (on a separate sheet) using a ruler.



Let us enumerate the three bodies with numbers 1, 2, and 3, according to the order in which they pass the leftmost point P shown in the figure. Let O_2 and O_3 denote the positions of the bodies 2 and 3, respectively at that moment when the body 1 is passing the middle point O. Similarly, let P_2 and P_3 denote the positions of the bodies 2 and 3, respectively at that moment when the body 1 is passing the leftmost point P. Let T denote the full period of motion of each of the bodies along this 8-shaped trajectory.

i) *(2 points)* Express the following travel times for one of the bodies: (a) from O_2 to O; (b) from O_3 to P_2.

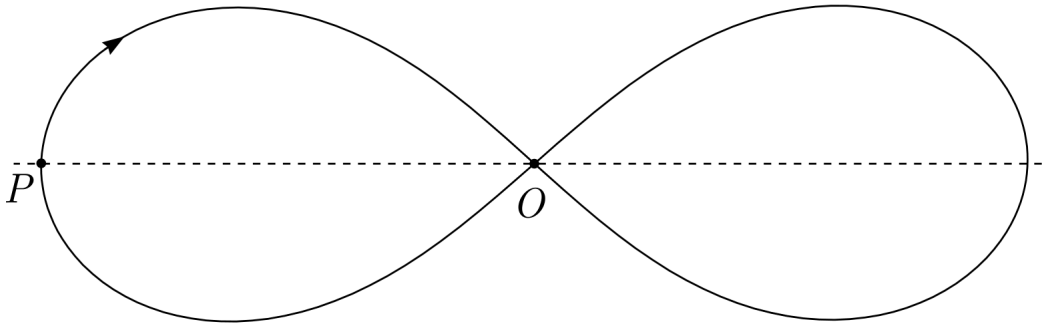
ii) *(1 point)* Let $\vec{v}_1$, $\vec{v}_2$, and $\vec{v}_3$ denote the velocities of the three bodies at a certain moment of time. Write down an equality relating these three vectors to each other.

iii) *(2 points)* Prove that the total angular momentum of this system is zero.

iv) *(2 points)* Construct the positions of the points O_2 and O_3 (use the figure on the separate sheet). Motivate your construction.

v) *(2 points)* Construct the positions of the points P_2 and P_3 (use the figure on the separate sheet). Find two independent ways of construction and motivate your methods.

vi) *(2 points)* Find the ratio of the speed of the bodies at point O to the speed at point P.



[1] Cristopher Moore, Phys. Rev. Lett. 70, 3675 (1993)