3. Motion in B (8 points) — *Andréas Sundström, Joonas Kalda (ii,iii).*

Particles with mass m and charge q are launched from the origin with speed v, parallel to the x-axis. There is a screen at $x = l$.

i) *(1 point)* The first particle is launched when there is an homogeneous electric field parallel to the x-axis, and there is no magnetic field. What should be the strength of the electric field so that the particle would never reach the screen?

ii) *(2 points)* Next, the electric field is turned off, a homogeneous z-directional magnetic field in region $l > x > 0$ is swithced on, and the second particle is launched. Knowing that particle's speed is just big enough to reach the screen, sketch the trajectory and find the magnetic fied strength B.

iii) *(2 points)* Finally, an electric field lying in the x, y-plane is turned on while B is kept unchanged. Third particle is fired from the origin, still parallel to the x-axis, but possibly with a different speed. It is observed that the particle travels without deviation. Further, the time it takes for it to reach the screen is the same as for the second particle. Find the magnitude of the electric field E.

iv) *(3 points)* Unrelated to the previous tasks, let us study now a weakly inhomogeneous magnetic field: the magnetic field lines have a radius of curvature which is much larger than the radius of curvature of the particle's trajectory. It appears that in that case, the so-called adiabatic invariant of a particle in magnetic field is conserved: the magnetic flux embraced by the particle's helix-shaped trajectory remains constant with an extremely good precision along the trajectory.

Consider a very simplified model for how a solar wind particles interact with the Earth's magnetic field. The strength of the Earth's magnetic field on its magnetic axis can be expressed as $B(z) = B_E\,(R_E/z)^3$, where $B_E = 3.12 \times 10^{-5}\,\mathrm{T}$ is the field strength at the Earth's surface on the magnetic pole, $R_E = 6370\,\mathrm{km}$ is the radius of the Earth and z is measured from the center of the Earth.

An electron of charge $-e = -1.60 \times 10^{-19}\,\mathrm{C}$ and mass $m_e = 9.11 \times 10^{-31}\,\mathrm{kg}$ is approaching the Earth with speed $u_0 = 500\,\mathrm{km/s}$ and hits the Earth's magnetic field right on its axis at a distance $R_0 = 5R_E$ with an angle α and starts spiraling towards the Earth.

If α is too large, the particle will be reflected by the increasing magnetic field strength as the particle approaches Earth. Find the condition on α for the particle to reach the surface of the Earth. You can neglect any gravitational and/or relativistic effects.