3. MOTION IN B (8 points) — *Andréas Sundström, Joonas Kalda (ii,iii).*

i) *(1 point)* In the homogeneous electric field of strength E along the x-axis, we can write down an electrostatic potential $\phi(x) = -xE$. For the particle *not* to hit the wall, the particle's kinetic enregy $mv^2/2$ must be less than $q\phi(l) = -qlE$, thus $|E| > \frac{mv^2}{2l|q|}$; the direction of E is such that $qE < 0$.

ii) *(2 points)* In the magnetic field, the particle moves along a circle with radius R such that the Lorentz force is equal to the centrifugal force, $qvB = mv^2/R$. Since the particle barely reaches the screen, the circular orbit must touch the screen. So $R = l$ and $B = \frac{mv}{lq}$.

iii) *(2 points)*

The first particle travels for a quarter period before stopping i.e. $t = \frac{T}{4} = \frac{\pi l}{2v}$. For the second particle the total Lorentz force must be zero so $u = \frac{E}{B}$. Equating times gives $\frac{l}{u} = \frac{\pi l}{2v}$ i.e. $E = \frac{2Bv}{\pi}$.

iv) *(3 points)* In order to derive the adiabatic invariant, we note that the magnetic flux of the helixal trajectory follows $\Phi \propto \text{Area} \times B_z \propto R^2 B_z \propto B_z/v_\perp^2$, where $v_\perp$ is the component of the velocity that's perpendicular to the magnetic field. Therefore the adiabatic invariant can be written as $v_\perp^2/B_z$.

During the motion of the electron, its kinetic energy is conserved because the magnetic field doesn't do any work. In the critical case, where the electron is almost reflected, the perpendicular compoment of the velocity of the electron at the surface of the earth will be equal to u_0. The adiabatic invariant then yields

$$\frac{v_{\perp 0}^2}{B(R_0)} = \frac{u_0^2}{B(R_E)}.$$

Now $v_{\perp 0} = u_0 \sin\alpha$ so the critical angle is given by

$$\alpha_0 = \arcsin\left(\sqrt{\frac{B(R_0)}{B(R_E)}}\right) = \arcsin\left(1/5\sqrt{5}\right) = 5.1^\circ.$$

The angle α has to be smaller than α_0 for the electron to reach the surface of the Earth.