4. Retroreflective film (12 points) — *Eero Uustalu and Jaan Kalda. (Solution: Taavet Kalda)*

i) *(2 points)* By shining the laser straight on to the retroreflective film, we see six dots appear on the screen. These dots represent the laser beam refracting through the six different prisms that appear on the film. In the symmetrical case, where $\alpha_i = \text{const.}$, we expect the dots to lie on the vertices of a hexagon. What we actually see is a hexagon, where two dots are squished inwards while the other four are equidistant from the centre. This implies that four angles and two angles are pairwise the same. We could say that faces 1 and 4 correspond to the dots that are squished inwards. In that case, $\alpha_1 = \alpha_4$, $\alpha_2 = \alpha_3 = \alpha_5 = \alpha_6$ and $\alpha_1 < \alpha_2$.

ii) *(2 points)* We can find the minimal deflection angles by holding the laser and the screen in place while tilting the film. In that case, the changing deflection angles are directly represented by the movement of the dots on the screen. Then the only thing left is to find the orientation of the film such that the deflection of the dots on the screen is minimal. Measurements yield $\beta_1 \approx 28°$ and $\beta_2 \approx 40°$.

iii) *(4 points)* Minimal deflection angle corresponds to the rays traversing the prisms symmetrically. This allows us to conveniently find the minimal deflection angle in terms of α and γ, where γ is the angle of the film with respect to the laser beam. Further measurements show $\gamma_1 = 11°$ and $\gamma_2 = 9°$. From geometry, we get $\beta/2 = \alpha/2 - \gamma$ so $\alpha = 2\gamma + \beta$. This gives $\alpha_1 = 50°$, $\alpha_2 = 58°$.

iv) *(1 point)* For our values, $\cos^2\alpha_1 + \cos^2\alpha_3 + \cos^2\alpha_5 = 0.97$. Either by trying different values or binary searching, we see that we can subtract $\Delta\alpha = 0.5°$ to get almost exactly 1 as the value for the sum of cosines squared.

v) *(3 points)* From Snell's law, $\sin(\alpha-\gamma) = n\sin(\alpha/2)$ so $n = \sin(\beta+\gamma)/\sin(\gamma+\beta/2) \approx 1.5$.