1. Phases of water (6 points) — *Solution by Johan Runeson, grading schemes by Johan Runeson and Adam Warnerbring.*

i) *(1.5 points)* We approximate the volume difference by the volume of the gas and use the ideal gas law: $V_g - V_l \approx V_g = \frac{nRT}{mp} = \frac{RT}{\mu p}$. Then it follows from the law of Clausius–Clapeyron that

$$\frac{\mathrm{d}p}{p} = \frac{\mu|\Delta H_{lg}|}{RT^2}\,\mathrm{d}T$$

which after integration gives

$$p = p_0 \exp\left(-\frac{\mu|\Delta H_{lg}|}{RT}\right),$$

where p_0 is a reference pressure. We also accept introducing a reference temperature T_0 so that

$$p = p_0' \exp\left[-\frac{\mu|\Delta H_{lg}|}{R}\left(\frac{1}{T} - \frac{1}{T_0}\right)\right], \tag{1}$$

where p_0' is another reference pressure.

Grading: Using ideal gas law — **0.5 pts**;
Writing correct differential equation — **0.2 pts**;
Solution has exponential dependence of $1/T$ — **0.6 pts**;
Correct solution overall — **0.2 pts**;

ii) *(1.5 points)* For any two points on the liquid–gas transision curve it holds that

$$\frac{p_2}{p_1} = \exp\left(-\frac{\mu|\Delta H_{lg}|}{R}\left[\frac{1}{T_2} - \frac{1}{T_1}\right]\right),$$

assuming that ΔH_{lg} is constant. Using for example $T_1 = 0\,^\circ\mathrm{C}$, $p_1 = 610\,\mathrm{Pa}$, $T_2 = 10\,^\circ\mathrm{C}$ and $p_2 = 1230\,\mathrm{Pa}$ (with temperatures converted to kelvin), we get $|\Delta H_{lg}| = 2503\,\mathrm{kJ/kg}$. Using this together with $T_3 = 15\,^\circ\mathrm{C} = 283.15\,\mathrm{K}$ and $T_4 = T_3 + 3\,\mathrm{K}$ gives

$$\frac{p_4 - p_3}{p_3} = \exp\left(-\frac{\mu|\Delta H_{lg}|}{R}\left[\frac{1}{T_4} - \frac{1}{T_3}\right]\right) - 1 = 0.21.$$

That is, the vapor pressure rises by **21%**. (This means that the water cycle will be enhanced, so that we can on average expect more humid weather after global warming. On the other hand, the Earth is not homogeneous, and in reality it is expected that wet locations become more wet while dry locations become more dry.)

Grading: Found ΔH_{lg} by measuring in graph — **0.4 pts**;
Numerical value for $|\Delta H_{lg}|$ correct within 10% — **0.3 pts**;
Correct formula for p_2/p_1 — **0.3 pts**;
Correct percentage $\pm 2\%$ — **0.5 pts**;

Grading for alternative solution: Extrapolation via derivative — **0.5 pts**;
Correct expression for final result — **0.5 pts**;
Correct percentage $\pm 2\%$ — **0.5 pts**;

iii) *(3 points)* First, look at the solid–gas transition line and assume also here that $V_g - V_s \approx V_g$. This gives a similar curve as for the liquid-gas transition but with a different transition enthalpy. From $T_5 = 0\,^\circ\mathrm{C}$, $p_5 = 610\,\mathrm{Pa}$, $T_6 = -10\,^\circ\mathrm{C}$ and $p_6 = 260\,\mathrm{Pa}$, we get the sublimation enthalpy $|\Delta H_{sg}| = 2828\,\mathrm{kJ/kg}$. This allows us to compute the melting enthalpy as

$$|\Delta H_{sl}| = |\Delta H_{sg}| - |\Delta H_{lg}| = 325\,\mathrm{kJ/kg}.$$

To measure the slope of the melting curve we draw a tangent in the origin and measure (for example) $\Delta T = 5\,\mathrm{K}$ and $\Delta p = -0.65 \times 10^8\,\mathrm{Pa}$. With $T = 273.15\,\mathrm{K}$, the law of Clausius–Claperyron finally gives

$$V_l - V_s = \frac{\Delta T}{\Delta p}\frac{|\Delta H_{sl}|}{T} = -9.2 \times 10^{-5}\,\mathrm{m^3/kg}.$$

(The experimental value is $-9.1 \times 10^{-5}\,\mathrm{m}^3/\mathrm{kg}$.) Note that ice has a larger volume than liquid water, which is an exception from most other substances.

Grading: Found ΔH_{sg} — **0.5 pts**;
Found ΔH_{sl} — **0.5 pts**;
Using slope of melting curve — **0.5 pts**;
Accurately measuring the slope of the melting curve near atmospheric pressure — **0.5 pts**;
Correct result within 50% — **0.5 pts**;
Correct result within 10% — **0.5 pts**;
Wrong sign — ***-0.5 pts***;