3. Conical room (3 points) — *Solution by Taavet Kalda, grading schemes by Maurice Zeuner, Eugen Dizer, and Titus Bornträger.*

If the distance from the base to the apex is H, then from energy conservation

$$gH = \frac{v_0^2}{2}.$$

Let the shortest distance from the base to the wall be h and the sought minimal speed v_1. From geometry, $h = H\sin\alpha$. Let's consider a new system of coordinates where the two axis x' and y' are parallel and perpendicular to the wall respectively. Gravitational acceleration has components $g_{x'} = g\cos\alpha$ and $g_{y'} = g\sin\alpha$. It is clear that the motion along the x' axis doesn't affect whether the projectile hits the wall. Because the motions in the x' and y' direction are independent, one has to set the component of $\vec{v}_1$ parallel to x' to $\mathbf{0}$ in order to minimize the total speed.

Then the problem reduces to hitting a projectile into a conventional ceiling of height h in effective gravity $g\sin\alpha$. Thus, from energy conservation,

$$g\sin\alpha\, h = gH\sin^2\alpha = \frac{v_1^2}{2}.$$

And so

$$v_1 = v_0\sin\alpha = \frac{\sqrt{3}}{2}v_0.$$

Grading: We expect to see mostly two different solution schemes. The first one is the given sample solution using the coordinate transformation. The second one is by mathematically deriving the intersection points of the trajectory with the walls.

Grading for sample solution:
Deriving the relation $gH = v_0^2/2$. — **0.5 pts**;
Using the relation $h = H\sin\alpha$. — **0.5 pts**;
Change of coordinate system and splitting the gravitational force — **1.0 pts**;
Further calculation — **0.5 pts**;
Correct result for v_1 — **0.5 pts**.

Grading for alternative methods:
Deriving the relation $gH = v_0^2/2$. — **0.5 pts**;
Equations of motion and derivation of the trajectory $y(x)$ of the projectile — **0.5 pts**;
Mathematical description of wall — **0.3 pts**;
Solving for intersection points and choosing the physical solution — **0.7 pts**;
Finding the optimal angle for minimum velocity (first derivative of velocity with respect to initial angle must be zero) — **0.5 pts**;
Correct result for v_1 — **0.5 pts**.