1. Photon rocket (5 points) — *Jaan Kalda, Oskar Vallhagen.*

Consider a hypothetical interstellar travel with a photon-propelled spaceship of initial rest mass $M = 1 \times 10^5$ kg. The on-board fuel (antimatter) is annihilated with an equal mass of matter to create photons yielding a reactive force. The matter required for annihilation is collected from the very sparse plasma of the interstellar space (assume that the velocity of the interstellar plasma is zero in the Earth's frame of reference). The speed of light is $c = 3 \times 10^8$ m/s.

i) *(1 point)* What should be the initial rate μ (kg/s) at which the antimatter should be burned for the acceleration to be equal to the free fall acceleration on Earth ($g = 9.81\,\mathrm{m/s^2}$)?

ii) *(3 points)* The engines of the spaceship are switched off when its rest mass has decreased down to $m_f = M/10$; what is its final speed?

iii) *(1 point)* The frequency of the emitted photons is measured by an observer on Earth. What is the frequency of the last photons (emitted just before the engine is switched off), as measured on Earth, if the frequency in the the space ship frame remains constant and equal to f_0?