3. Rotating space station (13 points) — *Jaan Kalda, Kaarel Hänni.*

A space station at a geostationary orbit has a form of a cylinder of length $L = 100\,\mathrm{km}$ and radius $R = 1\,\mathrm{km}$ is filled with air (molar mass $M = 29\,\mathrm{g/mol}$) at the atmospheric pressure and temperature $T = 295\,\mathrm{K}$ and the cylindrical walls serve as a ground for the people living inside. It rotates around its axis so as to create normal gravity $g = 9.81\,\mathrm{m/s^2}$ at the "ground".

i) *(0.5 points)* What is the rotation period τ?

ii) *(2 points)* A ball is thrown from a certain point on the "ground", and caught $t = \tau/2$ later at the very same point. What was the throwing speed of the ball? Neglect air drag.

iii) *(2 points)* A spherical balloon of radius $r = 3\,\mathrm{m}$ is filled with helium (molar mass $M' = 4\,\mathrm{g/mol}$), and is used to lift a weight of unknown mass m. The weight is fixed to the ball with a light rope of length $L = 20\,\mathrm{m}$, and the system rises until coming to a stop at the height $H = 500\,\mathrm{m}$ from the "ground". Determine the value of the mass m.

A rope of linear mass density $\lambda = 1\,\mathrm{kg/m}$ is fixed to the "ground" at two diametrically opposite points of the cylinder (so that the distance between the endpoints of the rope is $2R$). Let A, B, and C denote the two endpoints and the middle point of the rope, respectively.

iv) *(1.5 points)* Assuming that the height of the point C above the "ground" is h, determine $T_A - T_C$, the difference of the tension forces in the rope at the points A and C.

v) *(1.5 points)* Assuming that at the point A, the rope meets the "ground" at an angle α, determine the ratio of the tension forces T_A/T_C.

vi) *(1.5 points)* Find T_C if $h = 495\,\mathrm{m}$ by approximating the shape of the rope with a parabola.

vii) *(2 points)* The metallic walls of the space station carry a total charge Q. Inside the space station, a charged ball hovers above the "ground" motionlessly. Find the charge-to-mass ratio q/m of the ball. Ignore the effect of charges induced by the charged ball on the "ground".

viii) *(2 points)* Gauss theorem states that $\oint \vec{E} \cdot \mathrm{d}\vec{A} = 0$, where integral is taken over a closed surface embracing a volume V with no charges inside. How should this equality be modified by an observer on board of the space station if there are no other charges than the total charge Q distributed over the perimeter of the station?