4. Stretching gloves (8 points) — *Solution by Eero Uustalu, grading schemes by ....*

i) *(1 point)*

Cut a few strips of the material with the same constant width and mark them with two perpendicular lines close to the ends of the strip while leaving ample space for affixing or binding. Then roll one strip lengthwise into a cylindrical shape for stretching. Measure the initial length between the marked lines l_0. With a straight measuring tape affixed to the surface of the table, stretch one of the strips to the breaking point while determining the maximum length between the lines before the strip breaks $l_{\max}$. Repeat this at least three times to see if the data is reproducible. Calculate the result $\epsilon_{\max} = (l_{\max} - l_0)/(l_0)$.

In our measurement we got $\epsilon_{\max} = 5.5$.

Grading:

- Selection of the actual latex piece for the sample. Strips have to be of high quality and not include the finger end. **0.1 pts**
- Selecting the strip length: **0.2 pts**
  - at least 8cm: **0.2/0.2 pts**
  - at least 4cm: **0.1/0.2 pts**
  - shorter, not specified **0/0.2 pts**
  - *Remark.* If a series of different lengths of strips is used, then the medium value should be considered. Additionally, if the graph for finding $\epsilon_{\max}$ is used by including the point (0,0), then **0.1 pts** should be subtracted.
- Method of measurement, of which: **0.4 pts**
  - one end affixed with tape or held using a ruler **0.1 pts**.
  - lines marked on the strip, or the strip is affixed over the edge of the ruler **0.1 pts** (if both ends are affixed, **0.2 pts**, but can't get more than **0.4 pts** for this section).
  - using narrow strips **0.1 pts**.
  - measuring only the part of the strip that's being stretched (between marks) **0.1 pts**.
  - Measuring the full length of the stip **0.1 pts**.
- Repetition of measurements: **0.2 pts**
  - measurement repeated at least 5 times **0.2/0.2 pts**.
  - measurement repeated at least 3 times **0.1/0.2 pts**.
  - if distortion of measured $\epsilon_{\max}$ values is low **+0.1 pts**.
- Final numerical value. If the value lies in range 4-7 (depends on the material, even 3-11 may be ok) **0.1 pts**.

ii) (7 points)

Theoretical considerations.

We can assume the material has uniform thickness. Knowing that $\sigma = \frac{F}{A}$, $V = \text{const}$, and that any force applied to the strip affects all directions perpendicular to the applied force equally, then at any time for the same strip

$$V = a \cdot a \cdot k \cdot l = a_0 \cdot a_0 \cdot k \cdot l_0 = a_{\max} \cdot a_{\max} \cdot k \cdot l_{\max},$$

where a_0 is the initial thickness of material with no force applied and l_0 is the initial length of the strip with no force applied, a is the thickness when force F is applied and

l is the length of the strip when force F is applied, $a_{\max}$ and $l_{\max}$ are the respective values at the breaking force $F_{\max}$.

k is the ratio between the width d and the thickness a of the strip such that $d = k \cdot a$ where k remains constant for the strip while stretching. Therefore

$$\sigma = \frac{F}{a \cdot a \cdot k} \quad \text{and} \quad \sigma_{\max} = \frac{F_{\max}}{a_{\max} \cdot a_{\max} \cdot k}.$$

From $V = \text{const}$ we can deduce that $a \cdot a = \frac{a_0 \cdot a_0 \cdot l_0}{l}$ and $a_{\max} \cdot a_{\max} = \frac{a_0 \cdot a_0 \cdot l_0}{l_{\max}}$.

If we could apply the same force F on strips of different initial widths $d = k \cdot a$, we could vary the tension for each strip. Therefore we would be using different strips with different k values for each measurement

$$\sigma = \frac{F}{k} \cdot \frac{l}{l_0} \cdot \frac{1}{a_0 \cdot a_0} \quad \text{and} \quad \sigma_{\max} = \frac{F}{k_{\max}} \cdot \frac{l_{\max}}{l_0} \cdot \frac{1}{a_0 \cdot a_0},$$

where $k = d/a$ and $k_{\max} = d_{\max}/a_{\max}$ are the width to thickness ratios of the the respective strips used for F and $F_{\max}$. By combining the above, we get

$$\frac{\sigma}{\sigma_{\max}} = \frac{k_{\max}}{k} \cdot \frac{l}{l_{\max}},$$

but since $d_0 = k \cdot a_0$ is the initial unstressed width of one strip and $d_{\max 0} = k_{\max} \cdot a_0$ is the initial unstressed width of another strip stretched to the breaking point, for building the graph it transforms to:

$$\frac{\sigma}{\sigma_{\max}} = \frac{d_{\max 0}}{d_0} \cdot \frac{l}{l_{\max}} \text{ for the } Y \text{ axis and } \epsilon = \frac{l - l_0}{l_0} \text{ for the } X \text{ axis.}$$

Measurements.

We can assume the material has uniform thickness. We want to apply the same force on the strips with different initial widths.

Approach 1



We make many strips with different widths, mark them with the same initial length l_0 using perpendicular lines, measure and record the initial widths d_0 of each segment, then roll them lengthwise into a cylindrical shape and bind them one after another. The force will be the same for all the segments, but the value of σ for each segment will depend on the initial width of each individual strip.

A range where $d_{\min}$ and $d_{\max}$ differ at least 8 times is recommended.

Rolling the strip lengthwise into a cylindrical shape before binding gives the possibility to distribute the force evenly on all the strips regardless of the width of the strip!

Our solution was to make two identical strips with the smallest possible width and attach them to one another at one end to form a Y shape. The previously determined $\epsilon_{\max}$ was used to estimate the possible maximum stretching almost to the breaking point of the narrowest strip. In case of a breakage event the identical spare strip at the other end of the Y shape could be used for measurement. After affixing the multi-width rope of combined strips at its maximum stretch position, the values of l were

recorded for each individual section. Afterwards the rope was stretched till breaking to find the actual breaking length $l_{\max}$ of the narrowest strip.

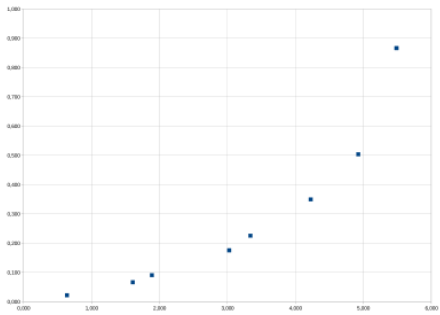
The **7 mm** wide initially **80 mm** long test strip broke at length **522 mm**. Therefore the measurement of the rope of strips was made using force F stretching the second **7 mm** wide **80 mm** test strip to **519 mm**.

But the **7 mm** stripe broke when it was stretched to **599 mm**.

So the $d_{\max 0} = 7\,\text{mm}$ and $l_{\max} = 599\,\text{mm}$.

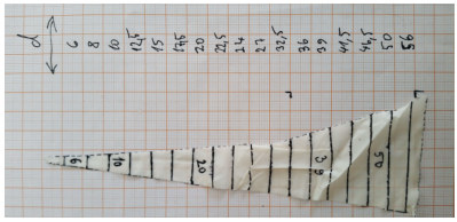
$(l - l_0)/l_0$	mm l_0	mm l	mm d_0	$\sigma/\sigma_{\max}$
0.638	80	131	71	0.022
1.606	80	208,5	37	0.066
1.888	80	231	30	0.090
3.025	80	322	21,5	0.175
3.338	80	347	18	0.226
4.225	80	418	14	0.349
4.925	80	474	11	0.505
5.488	80	519	7	0.866

And the graph, $\sigma/\sigma_{\max}$ for the Y axis and ϵ for the X axis.

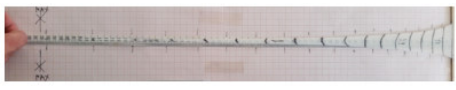


Approach 2

We make two identical elongated triangles from the longest piece of material available. We mark the triangles with evenly spaced perpendicular lines. (The lines are perpendicular to the central symmetry line of the elongated triangle.)



We record the initial length l_0 and the average width d_0 of each segment as defined by the perpendicular lines. When stretched, all segments will have the same force applied but each segment will have its own σ. We have to affix the base of the prolonged triangle to the ruler with tape (it has to be a good strong joint) and we affix the ruler to the table. A long strip of graph paper was taped to the table to have the rubber triangle stretched over it.



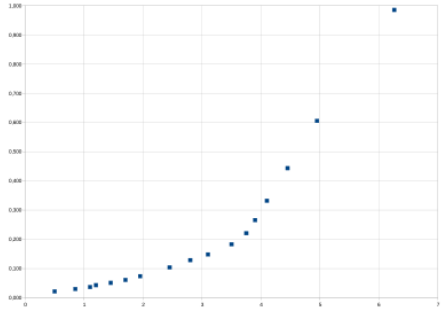
The triangle was stretched until the narrow end segment was near breaking point, and the values of l were recorded for each segment. (Most easily done by using a marker on the graph paper while holding the narrow end of the triangle at a fixed position with the other hand.) One of the triangles was actually stretched to the braking point, the other triangle was stretched to the breaking point only after the measurement was made.

The narrow end segment of the first triangle of length **10 mm** and of average width **6 mm** broke at length **73.1 mm**. Therefore the measurement of the second similar triangle tested was made using force F stretching the narrow end segment of the triangle to **72.6 mm**. The actual breaking of the narrow segment of the second triangle occurred at length **73.7 mm**.

So the $d_{\max 0} = 6\,\text{mm}$ and $l_{\max} = 73.7\,\text{mm}$.

$\sigma/\sigma_{\max}$	mm l_0	mm l	mm d_0	$(l - l_0)/l_0$
0.985	6	10	72.6	6.26
0.605	8	10	59.5	4.95
0.444	10	10	54.5	4.45
0.333	12.5	10	51	4.1
0.266	15	10	49	3.9
0.221	17.5	10	47.5	3.75
0.183	20	10	45	3.5
0.148	22.5	10	41	3.1
0.129	24	10	38	2.8
0.104	27	10	34.5	2.45
0.074	32.5	10	29.5	1.95
0.061	36	10	27	1.7
0.051	39	10	24.5	1.45
0.043	41.5	10	22	1.2
0.037	46.5	10	21	1.1
0.030	50	10	18.5	0.85
0.022	56	10	15	0.5

And the graph, $\sigma/\sigma_{\max}$ for the Y axis and ϵ for the X axis.



Grading:

GOOD practice:

- using the longest possible strips
- marking the initial length on the strip with a marker or
- measuring initial length to one straight affixed surface to another parallel affixed surface
- rolling the marker-marked strip lengthwise into a cylindrical shape for uniform stretching

- reasonable amount of repetitions (breaking is not very well repeatable)

BAD practice:

- intentionally variable length strips / short strips
- not marking the initial length on the strip with a marker
- not stretched with perpendicularly uniform strain
- not taking measurements only from actual part to be stretched
- not accounting for possible friction (some methods)

part ii

other solutions proposed:

- multiple similar masses used (not in the equipment list)
- a lever and some mass
- from the same latex material (a very long uniform and relatively thick strip is needed to be in proportional range, but then the measured force will be low)
- multiple similar strips in parallel as opposed to using fever strips in row and breaking with even force
- trigonometry and constant force
- different width strip pairs (force applied must remain constant for all pairs, in some solutions friction would need to be accounted for)

unsuccessful solving attempts / bad practices:

- using equipment not listed
- calculated, not measured (experiment!) **0 pts**.
- using a finger as a force measuring device **0 pts**.
- assuming the repeatedly applied force is constant (which it might not be) **0 pts**.
- measuring by varying the length of the strip while keeping the applied force constant (stress created remains constant) (force may be mg) **0 pts**.
- stretching the strip to different lengths not knowing the actual force applied **0 pts**.
- MOST OFTEN: Assumption that $F = kx$! That is actually an alternative lightly modified form of writing down the dependency WE HAVE TO MEASURE, not assume! (and the real dependency is not linear) **0 pts**.

Possible grading scheme for the solutions given as example:

part ii

theory:

- assuming that the material has initial uniform thickness **0.2 pts**
- idea that as $V = \text{const}$ then the dimensions in axes perpendicular to the applied force change in a fixed ratio to each other **1.0 pts**

- idea of changing σ by changing the width d of the material when constant force is applied **1.0 pts**
- idea of applying a common force to all the different samples at the same time **1.0 pts**
- finding how to calculate the value of ϵ to be plotted **0.2 pts**
- finding how to calculate the value of $\sigma/\sigma_{\max}$ to be plotted **0.6 pts**

measurements and data:

- the strips/intervals used are as long as possible (in our case at least 14cm) **0.1 pts**
- data range covered – value of the largest measured σ differs from the smallest measured σ at least 8 times **0.1 pts**
- data points – at least 8 points measured (**0.2 pts** for each correctly measured point or **0.1 pts** for each correctly measured point if less than 4 points measured (have you seen graph with 3 data points?), for maximum 8 measured points) MAX **1.6 pts**
- data evenly distributed **0.1 pts**
- at least one data point measured with a σ value at least 0,8 of $\sigma_{\max}$ **0.1 pts**
- measurements for the breaking event values correctly done **0.2 pts**
- chart of measured and calculated values given **0.3 pts**.

graph:

- graph correctly plotted with all measured points **0.3 pts**
- graph plotted in units asked (ratios, no units) **0.2 pts**

If the results of any intermediate calculations are excessively rounded, a deduction from the final score of $-$**0.1 pts** for every intermediate stage used.

using out of list equipment:

- If the out of list equipment used does not allow for any principally new methods of solution, any credit awarded for the parts of the solution that rely on the use of the out of list equipment or data measured with it will be reduced by 50%.
- If the out of list equipment does enable a principally new method of solution, credit will only be awarded for aspects of the solution that would be applicable in a solution that does not rely on the out of list equipment.