

2. Gas (6 points) — Solution by Jaan Kalda.

i) (2 points) Immediately after stopping, all the gas molecules move with the speed v . In the box's frame, the total energy of the molecules is conserved, so the average energy of each of the molecules is conserved, too: $\langle T \rangle = mv^2/2$. On the other hand, after thermalisation, this will be equal to $\frac{3}{2}kT$, hence $T = \frac{1}{3}mv^2/k = \frac{1}{3}\mu v^2/R$.

Grading: relating final average energy of the molecules to the initial energy in the box's frame or in another way showing that this energy gives rise to the temperature: 1 pt; relating this to the internal energy of monomolecular gas: 0.5 pt; expressing final answer: 0.5 pt. Solutions which used $\frac{1}{2}kT$ or kT instead of $\frac{3}{2}kT$ as the internal energy but otherwise correct get 1.5 pts.

ii) (2 points) Immediately after stopping, all the molecules move towards the wall with the same speed v . Hence, during time interval t , the molecules inside the near-wall region of thickness vt and volume $W = Avt$ (where A stands for the area of the wall) are hitting the wall. There are $N = \nu N_a W/V = \nu N_a Avt/V$ such molecules. After hitting, the molecules depart from the wall with the opposite velocity, hence each of them transfers momentum $\Delta p_0 = 2mv$ to the wall. So, the total transferred momentum is $\Delta p = N\Delta p_0 = 2\nu N_a Amv^2t/V$ which corresponds to the pressure $P = \Delta p/At = 2\nu N_a mv^2/V = 2\nu\mu v^2/V$.

Grading: expressing the number of molecules hitting a wall during time period t : 0.5 pts; finding the momentum transferred by each molecule: 0.5 pts; expressing the pressure in terms of the transferred momentum: 0.5 pts; bringing all these components together to get the final answer: 0.5 pts. Solutions that forget that particles obtain a velocity $-v$ after bouncing on the wall that are otherwise correct get 1.5 pts in total.

iii) (2 points) Fast molecules escape the trapping region, only molecules whose x -, y -, and z -components of velocity are smaller by modulus than $u = \frac{1}{2}V^{1/3}/\tau = 100$ m/s get trapped. This is much smaller than the thermal speed $v_T = \sqrt{RT/\mu} \approx 790$ m/s. In that range of velocities, the Maxwell distribution is almost constant, hence we may assume that the trapped molecules have velocity components evenly distributed from $-u$ to u . Hence,

$$\langle v_x^2 \rangle = \frac{1}{u} \int_0^u u^2 du = \frac{1}{3}u^2,$$

so that the total average kinetic energy is $\frac{1}{2}m(v_x^2 + v_y^2 + v_z^2) = \frac{1}{2}\mu u^2$. On the other hand, this is equal to $\frac{3}{2}kT$, hence $T = \frac{1}{3}\mu u^2/k = \frac{1}{3}\mu u^2/R \approx 1.6$ K.

Grading: realising that only slow molecules get trapped: 0.3 pts; finding the maximal velocity projection of trapped molecules: 0.3 pts; noting that velocity distribution function of trapped molecules is a constant: 0.3 pts; finding $\langle v_x^2 \rangle$: 0.3 pts; expressing hence the kinetic energy: 0.2 pts; relating this to $\frac{3}{2}kT$: 0.3 pts bringing all these components together to get the final answer: 0.3 pts.