3. Rocket (5 points) — *Solution by Jaan Kalda.*

i) *(1 point)* Momentum of the reflected photons was $p_0 = W/c = \alpha M_0 c$, and becomes after reflection opposite, $p_1 = -p_0 = -\alpha M_0 c$, hence the momentum transferred to the rocket $\Delta p = 2\alpha M_0 c$. Due to the conservation of the total momentum, the momentum of the rocket $M_0 v = \Delta p = 2\alpha M_0 c$, hence $v = 2\alpha c$.

Grading: relationship $p_0 = W/c$: 0.3 pts; $\Delta p = 2p_0$: 0.2 pts; $M_0 v = \Delta p$: 0.4 pts; final answer: 0.1 pts.

ii) *(2 points)* Let the final relativistic mass of the rocket be M, and the momentum — p, and let the total mass of photons after reflection be μ. For convenience, we'll be using units by which $c = 1$. Then we have the relativistic invariant for the 4-momentum

$$p^2 + M_0^2 = M^2.$$

As the rest mass of photons is zero, the time- and space components of their 4-momentum are equal, hence the momentum of the photons $P = \mu$. The momentum conservation is written as

$$p = M_0 + \mu;$$

the energy conservation law is written as

$$M + \mu = 2M_0.$$

The last two equations yield $p = 3M_0 - M$. Upon taking this equation into square and combining with the first equation, we obtain $M = \frac{5}{3}M_0$, hence $\mu = 2M_0 - M = \frac{1}{3}M_0$, and $p = M_0 + \mu = \frac{4}{3}M_0$. Therefore, the speed of the rocket $v = p/M = \frac{4}{5}$. Returning to the SI system of units, this corresponds to $v = \frac{4}{5}c$.

Grading: Correctly written conservation laws (momentum, energy): 0.5 pts each; relativistic invariant for the rocket: 0.3 pts; equality $p = \mu$: 0.3 pts; solving the obtained set of equations to find the relativistic mass (energy) of the rocket: 0.1 pts; and expressing the final answer for the speed: 0.3 pts. Partial credit if the speed has not been found: 0.2 pts for expressing $v = p/M$.

iii) *(2 points)* The perceived acceleration is proportional to the transfer rate of the momentum, from photons to the rocket, in the rocket's frame. This transfer rate is inversely proportional to the time interval between two subsequent photons, and to the momentum of each of the photons. Due to the Doppler shift, both get longer with the increasing speed by the Doppler factor

$$k = \sqrt{\frac{1+v}{1-v}}.$$

Hence, the perceived acceleration is proportional to k^{-2}. When the rocket is still at rest, this factor is equal to 1; at the end of the process, it is equal to

$$\frac{1-v}{1+v} = \frac{1}{9}.$$

Therefore, the acceleration is reduced 9 times.

Grading: Stating the two reasons why acceleration becomes smaller: 0.4 pts each. Using the Doppler effect formula for finding the red shift of the photons in the rocket's frame: 0.4 pts; using the Doppler effect formula for finding the time delay between two subsequent photons: 0.6 pts; expressing the final answer: 0.2 pts.