4. AC FILTER (5 points) — *Solution by Jaan Kalda.*

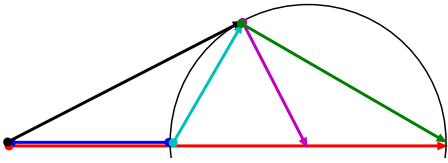
i) *(2 points)* The output voltage V_{out} is the difference of the voltages on the capacitors. Hence, for V_{out} to become infinite, one of the currents must become infinite. This is possible only if the impedance of the lower branch becomes zero:

$$\frac{1}{\mathrm{i}\omega_0 C_0} + \mathrm{i}\omega_0 L = 0,$$

hence $\omega_0 = 1/\sqrt{LC_0}$.

Grading: Concluding that the impedance of the lower branch must be zero: 0.8 pts; expressing this impedance: 1 pt; finding the final answer: 0.2 pts.

ii) *(3 points)* Let us draw a phasor for this circuit. As compared with $\omega = \omega_0$, the ratio of the impedances on the inductor and on a capacitor is increased four-fold. For the lower branch, these two impedances were equal previously, hence now the impedance of the inductor is four times bigger than the impedance of the capacitor; the same applies to the corresponding voltages: $V_L = 4V_{C0}$. The two voltage vectors are antiparallel and must add up to the input voltage V_0, hence $V_L - V_{C0} = V_0$, hence $V_{C0} = V_0/3$ and $V_L = \frac{4}{3}V_0$. The voltage vectors on C_1 and R are perpendicular to each other and must add up also to the input voltage V_0, hence these three voltage vectors form a right triangle. According to the Thales theorem, the right angle must lie on a circle, with the input voltage being a diameter of this circle. This is depicted in the figure below where the voltage vectors are color-couded as follows: output — black; capacitor C_1 — cyan; capacitor C_0 — blue; inductor — red; resistor — green. Radius of the circle is shown in purple.



From this figure, it becomes obvious that the phase shift φ is maximal when the output voltage vector is tangent to the circle, hence

$$\varphi = \arcsin\frac{V_0/2}{V_L - V_0/2} = \frac{3}{5},$$

and

$$V_{\text{out}} = \frac{V_0/2}{\tan\varphi} = \frac{2}{3}V_0.$$

Alternatively, the problem can be solved using the standard impedance-based approach, but this will be mathematically significantly more technical.

Grading: Concluding that $V_{C0} = V_0/3$ and $V_L = \frac{4}{3}V_0$: 0.5 pts; concluding that the potential of the upper output node draws a circle (applying the Thales theorem): 1 pt; noting that phase shift is maximal when the output voltage vector is tangent to the circle: 1 pt; finding the phase shift: 0.2 pts; finding V_{out}: 0.3 pts.