4. Totality (8 points) — *Taavet Kalda, Jaan Kalda.*

Total solar eclipses are a rare phenomenon which occur when the Moon completely covers the disk of the Sun for some parts of the Earth. This doesn't happen during every solar eclipse because the Moon's apparent size in the sky is sometimes too small to fully cover the Sun, but also because the Moon's shadow usually misses the Earth due its orbital inclination. As a result, total solar eclipses occur on average every 18 months.

Let us consider a total solar eclipse where during the peak, the centre-points of Earth, the Moon and the Sun lie on a line on the same plane as the equator. We measure that right before the total solar eclipse ends at latitude $\lambda = 28.5°$, the totality lasts for $t_0 = 2\,\text{min}$. Earth's radius is $r_e = 6370\,\text{km}$, Moon's radius is $r_m = 1740\,\text{km}$, orbital period of the Moon $T_m = 27.3\,\text{d}$, orbital radius of the Moon $R_m = 384\,000\,\text{km}$. One day on Earth is $T_0 = 24\,\text{hrs}$.

i) *(1.5 points)* For how long is there a place on Earth where the total solar eclipse is observable?

ii) *(1 point)* How many degrees in longitude on Earth does the total solar eclipse cover?

iii) *(1.5 points)* What is the width of the path of totality near the equator?

iv) *(1.5 points)* What is the longest amount of time the total eclipse is visible for a single location on Earth?

v) *(1 point)* For how long is the total eclipse near the location described in iii), at the distance of $a = 50\,\text{km}$ from the centreline of the eclipse path?

vi) *(1.5 points)* Find the average time interval between two total solar eclipses for a given location on Earth by making the following simplifying assumptions:

a) the average width of the full eclipse path is equal to the arithmetic average of its smallest and largest width;

b) typical width of a full eclipse path is half of the average width of the eclipse studied above;

c) typical length of a full eclipse path is equal to the length of the eclipse path studied above if the Earth were not rotating;

d) total solar eclipses occur with equal likelihood anywhere on Earth.