2. Oklo Fission Reactor (7 points) — *Solution by Tudor Plopeanu, Jaan Kalda and Topi Löytäinen.*

i) *(1.5 points)* The rate of natural decay is proportional to the amount of decaying material. If ν is the amount of ^{235}U at some point in time, then $\frac{\mathrm{d}\nu}{\mathrm{d}t} = -k\nu$ for some positive constant k. Then, $\frac{\nu_b}{\nu_a} = \mathrm{e}^{-k(t_b-t_a)}$ for some moments $t_b > t_a$. The half-time is defined as the

time until the amount of material is halved, so $k\tau_5 = \ln 2$, $k = \frac{\ln 2}{\tau_5}$, $\frac{\nu_b}{\nu_a} = (e^{\ln 2})^{\frac{t_b-t_a}{\tau_5}} = 2^{\frac{t_b-t_a}{\tau_5}}$. Therefore, the amount of ^{235}U in natural uranium when the Oklo's reactor operated was $2^{\frac{T_0}{\tau_5}}$ times higher. Similarly, the amount of ^{238}U was $2^{\frac{T_0}{\tau_8}}$ higher. The natural abundence of ^{235}U T_0 years ago was

$$R_1 \approx \frac{\frac{\nu_5}{\nu_8} 2^{\frac{T_0}{\tau_5}}}{\frac{\nu_5}{\nu_8} 2^{\frac{T_0}{\tau_5}} + 2^{\frac{T_0}{\tau_8}}} \approx 3.16\%,$$

where ν_5 and ν_8 represent the number of isotopes of uranium. Keep in mind that $R = \nu_5/(\nu_5 + \nu_8)$, so $\nu_5/\nu_8 = R/(1-R)$.

Grading: (preliminary)

1. Amount of ^{238}U during operation $\nu_8' = \nu_8 2^{T_0/\tau_8}$. **(0.3 pts)**

2. Amount of ^{235}U during operation $\nu_5' = \nu_5 2^{T_0/\tau_5}$. **(0.3 pts)**

3. Abundance of ^{235}U during operation expressed in terms of ν_5 and ν_8. **(0.3 pts)**

4. Ratio ν_5/ν_8 expressed in terms of R. **(0.3 pts)**

5. Abundance of ^{235}U expressed in terms of R and numerical answer. **(0.3 pts)**

ii) *(2 points)* From here on we shall assume that before the Oklo reactor started operating, its abundence of uranium isotopes matched the natural abundence. After it finished operating, the ^{235}U isotopes kept decaying at the normal rate, reaching the value we see today.

The mass of ^{235}U after the reactor finished operating was $M_1 \approx 2^{\frac{T_0}{\tau_5}} M R_O \approx 1.84 \times 10^7$ kg.

The ^{238}U isotopes underwent normal decay, and we shall use them to compute the mass of ^{235}U before the reactor started. The mass of ^{238}U was approximately $M(1-$

$R)2^{\frac{T_0}{\tau_8}}$. The mass of ^{235}U before the reactor operated can, as such, be estimated as

$$M_0 = \frac{R_1}{1-R_1} M(1-R)2^{\frac{T_0}{\tau_8}} \approx 2.14 \times 10^7 \text{ kg}.$$

The number of ^{235}U nuclei which reacted is $\nu = [(M_0 - M_1)/0.235] \cdot N_A \approx 7.61 \times 10^{30}$. The amount of energy generated through fission is the above value multiplied by E_0. The average power of the reactor is $\nu E_0/T \approx 7.73 \times 10^7$ W.

Grading: (preliminary)

1. Mass of ^{235}U by the end of operation $M_5' = 2^{T_0/\tau_5} M R_0$. **(0.4 pts)**
2. Mass of ^{238}U by the end of operation $M_8' = 2^{T_0/\tau_8} M(1-R)$. **(0.3 pts)**
3. Mass of ^{235}U at the beginning of operation $2^{T_0/\tau_8} M(1-R)\frac{R'}{1-R'}$. **(0.3 pts)**
4. Number of atoms that have undergone fission $N = N_A \Delta M/0.235$. **(0.3 pts**
5. Energy $E = NE_0$. **(0.2 pts)**
6. Units converted correctly. **(0.2 pts)**
7. Power: expression and numerical value. **(0.3 pts)**

iii) *(1.5 points)* Neutrons released from fission tend to have a high kinetic energy which have a low probability to cause further fissions. We need to slow them down to increse this probability. Water is relatively good for slowing down neutrons. This slowing down is called "moderation". The reactor could not explode since the water necessary for moderation would vapourize if the power increased too much. This would prevent an uncontrolled chain reactor from happening. The reactor's power depends on water being present and thus the operation was self-regulating.

When the water intake doubled, the amount of power doubled.

Grading: (preliminary)

1. Idea that neutrons from fission unlikely to cause fission unless moderated: **(0.3 pts)**

2. Idea of water as a moderator: **(0.2 pts)**

3. Idea of self-regulating behavior: **(0.5 pts)**

4. Observation that power doubles when flow doubles: **(0.5 pts)**

iv) *(2 points)* The amount of energy generated by the reactor was νE_0. We shall consider the vast majority of it to have gone into heating up and vaporizing water. Let us consider water flowing in at $0°C$ and leaving at $100°C$. Then, the amount of energy absorbed by 1 kg of water is $(100° K)c + L = E_w = 2.68 \times 10^6$ J. The total mass of water that flowed into Oklo's reactor is $\nu E_0/E_w = 9.09 \times 10^{13}$ kg.

Grading: (preliminary)

1. Approximation that all energy went into water **(0.5 p)**

2. $\Delta T \approx 100$ **(0.3 p)**

3. Both heating and vaporization considered **(0.3 p)**

4. $E_w = 2.68 \times 10^6$ J or similar idea **(0.3 p)**

5. $\nu E_0/E_w = 9.09 \times 10^{13}$ kg **(0.6 p)**