7. WAVES (4 points) — *Solution by Tudor Plopeanu.*

i) *(1 point)* Both gk^α and $\frac{\sigma}{\rho}k^\beta$'s dimensionalities are s^{-2}, while k's dimensionality is m^{-1}. The surface tension's dimensionality is kg /s^2. As such, $\alpha = 1$ and $\beta = 3$.

Grading: (preliminary)

1. Dimensional analysis (or similar technique). **(0.2 pts)**
2. Units of σ **(0.2 pts)**
3. $\alpha = 1$. **(0.3 pts)**
4. $\beta = 3$. **(0.3 pts)**

ii) *(3 points)*

The phase and group velocities of the wake are equal, i.e. $\frac{\mathrm{d}\omega}{\mathrm{d}k} = \frac{\omega}{k}$. We can find $\frac{\omega}{k}$ from the Mach angle formed by the wave front in the picture: $\sin\mu = \frac{\omega}{Uk}$. Differentiating the dispersion relation,

$$2\omega\frac{\mathrm{d}\omega}{\mathrm{d}k} = (g + 3\frac{\sigma}{\rho}k^2) \implies 2\omega^2 = k(g + 3\frac{\sigma}{\rho}k^2).$$

Substituting our dispersion relation, we obtain $g = \frac{\sigma}{\rho}k^2$ and $\omega^2 = 2gk$. Substituting $g^{\frac{1}{2}} = (\frac{\sigma}{\rho})^{\frac{1}{2}}k$, we reach $\frac{\omega^2}{k^2} = 2(\frac{g\sigma}{\rho})^{\frac{1}{2}}$. Thus, we can find

$$\sigma = \frac{\rho U^4 \sin^4\mu}{4g} \approx 60\,\mathrm{g\,s^{-2}}.$$

for measured $\mu \approx 21.5°$.

Grading: (preliminary)

1. Identifying $d\omega/dk = \omega/k$. **(0.6 pts)**
2. Correct angle i.e. $\sin(\mu) = \omega/(Uk)$ **(0.6 pts)**
3. Measuring μ from the picture **(0.4 pts)**
4. Value $\mu \in [20°, 30°]$ **(0.3 pts)**
5. Calculate the derivative $d\omega/dk$ **(0.4 pts)**
6. Correct expression for σ **(0.4 pts)**
7. Value $\sigma \approx 60\,\mathrm{g\,s^{-2}}$ **(0.3 pts)**