1. Flying Dumbbell (10 points) — *Jaan Kalda.*

In this problem, we shall study the dynamics of a dumbbell consisting of two steel balls, each with radius $r = 1\,\mathrm{cm}$, connected by a steel rod with diameter $d = 1\,\mathrm{mm}$ and length $l = 10\,\mathrm{cm}$, in the absence of gravity. Unless instructed otherwise, assume steel is perfectly elastic. You may simplify your calculations by assuming $l \gg r$.

i) *(2 points)* Given that the Young's modulus of steel is $Y = 2\times10^{11}\,\mathrm{Pa}$ and the density of steel is $\rho = 7800\,\mathrm{kg\,m^{-3}}$, determine the period T of free longitudinal oscillations of the dumbbell. (Do not consider oscillations with standing waves in the rod where the balls remain almost at rest.) Young's modulus is the ratio of a material's stress (force per unit area) to its strain (relative deformation).

ii) *(2 points)* Estimate the impact time τ when a steel ball bounces off a steel wall.

iii) *(2 points)* The dumbbell moves toward a steel wall with velocity $\vec{v} = -v\hat{x}$, with its axis perpendicular to the wall, and bounces back. Here, $\hat{x}$ denotes a unit vector along the axis perpendicular to the wall. Sketch how the x-component v_x of the front ball's velocity (the ball that collides with the wall) depends on time.

iv) *(2 points)* Now, the dumbbell moves toward a steel wall with velocity $\vec{v} = -v\hat{x}$ as before, but its axis forms an angle α with the x-axis. The interaction of the front ball with the wall depends qualitatively on the angle α, with a transition from one type of interaction to another occurring at $\alpha = \alpha_0$. Determine the value of α_0. *Hint:* $\min\left(\frac{\sin x}{x}\right) \approx -0.217$.

v) *(2 points)* Under the assumptions of the previous task, let $\alpha > \alpha_0$. Additionally, assume that while steel is highly elastic, it is not infinitely so: any oscillations excited in the rod will decay by the time the rear ball collides with the wall. Determine the speed with which the centre of mass of the dumbbell departs from the wall.