3. Nuclear Reactors (6 points) — *Topi Lind.*

In order to maintain a chain reaction in a modern thermal-neutron nuclear reactor one needs three things: 1. nuclear fuel (e. g. U^{235}), 2. moderator (e. g. water) and 3. coolant. In most cases the moderator acts as the coolant as well. Neutrons released from a thermal fission of U^{235} have a mean kinetic energy of approximately $E_0 = 2\,\mathrm{MeV}$. However, neutrons which are that fast are inefficient in triggering fission of U^{235}: neutrons need to be slowed down to an average kinetic energy of $E_f = 0.025\,\mathrm{eV}$. In what follows, justify why non-relativistic approximations can be used unless explicitly instructed otherwise.

i) *(1 point)* The rest energy of neutrons $m_\mathrm{n}c^2 = 940\,\mathrm{MeV}$, the Boltzmann constant $k_\mathrm{B} = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$, and the elementary charge $e = 1.602 \times 10^{-19}\,\mathrm{C}$. What is the required speed of neutrons, i. e. the speed of neutrons with kinetic energy E_f? What is the temperature T_f of a neutron gas where the average kinetic energy of neutrons is E_f?

ii) *(1 point)* What is the initial speed of neutrons, i. e. the speed of neutrons with energy E_0?

iii) *(2.5 points)* From a completely non-relativistic point of view, what should be the mass of the moderator's atoms to slow down the fast neutrons as efficiently as possible? If the mass of the moderator's atoms were to be $M = 135\,m_\mathrm{n}$, how many collisions with such atoms at a temperature much lower than T_f would a fast neutron need to experience to slow down from E_0 to E_f? Assume that all collisions are elastic and central.

iv) *(1.5 points)* Nuclear fuel, i.e. U^{235}, is placed inside metal rods and pressurized with helium gas to $p_0 = 2.5\,\mathrm{MPa}$. During operation, as U^{235} keeps on fissioning inside the fuel rods, there is a build up of gas inside the rods. With a non-invasive ultrasound measurement we can measure that the gas pressure inside the rod after it is finally picked out from the core is $p = 6.5\,\mathrm{MPa}$. Assuming that the gas released inside the rods is completely made of xenon isotope $^{135}_{54}\mathrm{Xe}$ and that the initial gas volume drops from $V_0 = 18\,\mathrm{cm^3}$ to $V = 9\,\mathrm{cm^3}$ due to the swelling of the fuel pellets, how many moles of xenon are released from fission? What is the ratio of helium to xenon inside the rod? The measurements are done at $T_0 = 20\,^\circ\mathrm{C}$; the universal gas constant $R = 8.31\,\mathrm{J\,mol^{-1}\,K^{-1}}$.