3. Nuclear Reactors (6 points) — *Solution and grading scheme by Topi Lind, Melvin Storbacka, Oleg Kosik, Aleksandr Sorokin and Ludmila Belogrudova.*

i) *(1 point)* If the speed of the particle is much less than the speed of light, we can use non-relativistic approach. For non-relativistic particles we know $v = \sqrt{2E_k/m}$. Substituting values gives us $v_f = 2.2 \times 10^3\,\mathrm{m\,s^{-1}}$. This

is much less than the speed of light and thus justified. Another way to justify the applicability of the non-relativistic approach would be to say that kinetic energy is significantly less than the rest energy ($E_f \ll m_\mathrm{n}c^2$).

Typo in problem description. $E_f = 0.025\,\mathrm{eV}$ is the mode of the Maxwell–Boltzmann distribution which gives $E = k_bT$. Average gives $E = (3/2)k_bT$.

Using the mode of the Maxwell–Boltzmann distribution, $E = k_\mathrm{B}T$, and remembering to convert from eV to J correctly, we find

$$T_f = \frac{0.025 \cdot 1.602 \times 10^{-19}}{1.38 \times 10^{-23}} = 290\,\mathrm{K}.$$

With $E = (3/2)k_\mathrm{B}T$ we find $T_f \approx 193\,\mathrm{K}$.

ii) *(1 point)* The non-relativistic approach justified as in previous task. Same approach gives us $v_0 = 2.0 \times 10^7\,\mathrm{m\,s^{-1}}$.

Grading i)+ii): (preliminary)

- Expresses $v_f = \sqrt{2E_f/m}$ and/or $v_0 = \sqrt{2E_0/m}$ **0.3 pts**
- Calculates $v_f = 2.2 \times 10^3\,\mathrm{m\,s^{-1}}$ and/or $v_0 = 2.0 \times 10^7\,\mathrm{m\,s^{-1}}$ for two correct numerical values; **0.5 pts**; if only one value is correct **0.3 pts**.
- Uses $E_f = \frac{3}{2}k_BT$ **0.3 pts**
- Using this formula calculates $T = 193\,\mathrm{K}$ **0.3 pts**
- Justifies the validity of the classical approach in both cases **0.3 + 0.3 pts**
  Remark. Using $E_f = k_BT$ without justification and finding $T = 290\,\mathrm{K}$ gives **0 pts** for the formula and **0.3 pts** for the numerical calculation.

iii) *(2.5 points)* In a collision between particle 1 ($m_1, v_{1,i}$ and $v_{1,f}$) and particle 2 ($m_2, v_{2,i}$ and $v_{2,f}$) momentum is conserved,

$$m_1(v_{1,f} - v_{1,i}) = m_2(v_{2,i} - v_{2,f})$$

and since the collisions are elastic, kinetic energy is also conserved:

$$m_1(v_{1,f}^2 - v_{1,i}^2) = m_2(v_{2,i}^2 - v_{2,f}^2).$$

Dividing the latter by the former leads to

$$v_{1,i} + v_{1,f} = v_{2,i} + v_{2,f}.$$

Substituting this to the conservation of momentum gives for particle 1:

$$v_{1,f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1,i} + \frac{2m_2}{m_1 + m_2} v_{2,i},$$

and similarly for particle 2:

$$v_{2,f} = \frac{2m_1}{m_1 + m_2} v_{1,i} + \frac{m_1 - m_2}{m_1 + m_2} v_{2,i}.$$

We see that with $m_1 = m_2$ there is a maximum transfer of momentum. Assuming that particle 1 is the neutron and particle 2 is the target, and that the target is at rest for all intents and purposes, the mass of the moderators atoms should be the same as the neutrons.

In a single collision with a stationary atom of the moderator, the speed of the neutron decreases by the factor of $\frac{m_1 - m_2}{m_1 + m_2}$, so the speed of the neutron after N head-on collisions with stationary atoms of moderator will be

$$v_N = v \left(\frac{m_1 - m_2}{m_1 + m_2} \right)^N .$$

Hence,

$$\begin{aligned} N &= \frac{\ln(v_f / v_0)}{\ln[(m_1 - m_2)/(m_1 + m_2)]} \\ &= \frac{1}{2} \frac{\ln(E_f / E_0)}{\ln[(m_1 - m_2)/(m_1 + m_2)]} = 614. \end{aligned}$$

Grading: (preliminary)

- $T \ll T_f$, so moderator atoms are essentially at rest **0.3 pts**
- Justifies that maximum momentum transfer is when $m_{\mathrm{n}} = M$ **0.4 pts**
- Applies energy and momentum conservation **0.3 pts**+ **0.3 pts**
- Expresses $u = v \frac{m_1 - m_2}{m_1 + m_2}$ **0.4 pts**
- Expresses $v_f = v_0 \left(\frac{m_1 - m_2}{m_1 + m_2} \right)^N$ **0.5 pts**
- Calculates $N = 614$ **0.3 pts**

iv) *(1.5 points)* We can model the gas inside the rod as an ideal gas. Simply due to swelling the pressure inside the rod would increase from 2.5 MPa to 5 MPa as we know from Boyle's law $P_1V_1 = P_2V_2 \rightsquigarrow P_2 = P_1V_1/V_2$. Thus, the release of xenon must contribute 1.5 MPa's worth of pressure due to Dalton's law $p_{\mathrm{tot}} = p_{\mathrm{He}} + p_{\mathrm{Xe}}$. From ideal gas law we find the amount of xenon moles as

$$n_{\mathrm{Xe}} = p_{\mathrm{Xe}}V/RT_0 = 5.5 \times 10^{-3}\,\mathrm{mol},$$

where $p_{\mathrm{Xe}} = 1.5\,\mathrm{MPa}$, $V_2 = 9\,\mathrm{cm}^3$ and $T = 293\,\mathrm{K}$. In a similar manner we find that the amount of helium in the beginning was

$$n_{\mathrm{He}} = p_{\mathrm{He}}V_0/RT_0 = 1.8 \times 10^{-2}\,\mathrm{mol},$$

where $P_{\mathrm{He}} = 2.5\,\mathrm{MPa}$, $V_0 = 18\,\mathrm{cm}^3$ and $T_0 = 293\,\mathrm{K}$. The ratio of the two is

$$\frac{n_{\mathrm{He}}}{n_{\mathrm{Xe}}} \approx 3.3.$$

Grading: (preliminary)

- Applies Boyle's law **0.3 pts**
- Applies Dalton's law **0.4 pts**
- Expresses $n_{\mathrm{Xe}} = p_{\mathrm{Xe}}V/RT_0$ **0.2 pts**
- Calculates $n_{\mathrm{Xe}} = 5.5 \times 10^{-3}\,\mathrm{mol}$ **0.2 pts**
- Expresses $n_{\mathrm{He}} = p_{\mathrm{He}}V_0/RT_0$ **0.2 pts**
- Calculates $\frac{n_{\mathrm{He}}}{n_{\mathrm{Xe}}} = 3.3$ **0.2 pts**