4. Rod and bead (8 points) — *Eero Ristolainen, Jaan Kalda.* A bead is threaded on a rigid frictionless rod. The rod rotates with constant angular velocity ω about a horizontal axis that is perpendicular to the rod and passes through a fixed point on it. Gravitational acceleration is g.

i) *(1 point)* With suitable initial conditions, the trajectory of the bead is a circle. Sketch this trajectory in the plane perpendicular to the rotation axis, in a coordinate system with the axis at the origin.

ii) *(1 point)* Given that the trajectory is exactly a circle, find its radius.

iii) *(2 points)* The circular trajectory of the previous tasks is unstable. To stabilise it, one attaches a spring that produces a restoring force $-kx$ when the bead is displaced by x along the rod from the axis of rotation. For what values of the stiffness k can the bead move along a *stable* circular trajectory, and what is the radius of such a trajectory?

iv) *(4 points)* Next, the spring is removed, the bead is given a charge q, and a uniform electric field of strength E whose vector rotates with angular velocity Ω in the same direction as the rod is introduced. With suitable initial conditions, the bead follows the trajectory shown below. Find Ω, g, and the ratio $a_E = Eq/m$, where m is the mass of the bead, in terms of ω and d, the side length of the grid squares shown in the figure. You may take measurements from the figure.

