1. Rotating Cylinder

(a) [1] Alice shows Bob her new cylinder collection. One of them is a hollow cylinder. The cylinder has length L, radius r and wall thickness $\delta \ll r$ for both the bases and the curved surface. The mass density of the cylinder is ρ. Find the mass m of the cylinder and the moment of inertia I of the cylinder about its axis.

(b) [2] The cylinder is made of carbon nanotubes and has high tensile strength, $\sigma = \frac{F_{max}}{A}$, where F_{max} is the maximum tension force the cylinder can sustain without breaking and A is the cross-sectional area perpendicular to the direction the force is applied (see Fig. 1-1). By considering an element of the cylinder when spinning (refer to the top view of the spinning cylinder in Fig 1-2), estimate the maximum angular velocity ω_{max} at which the cylinder can spin before breaking. Express in terms of σ, ρ and r. Give a numerical value of ω_{max} for $L = 1m$, $r = 10cm$, $\delta = 50\mu m$, $\sigma = 20GPa$, $\rho = 1.3g/cm^3$. Ignore deformation. You may assume the sides break before the bases do.

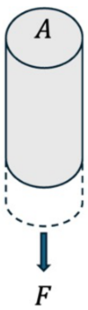
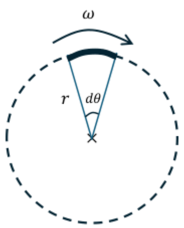


Fig 1-1



Top view
Fig 1-2

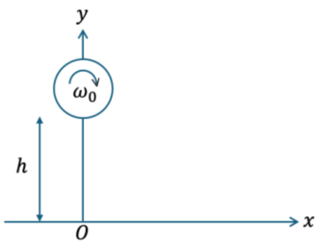


Fig 1-3

In what follows, we shall assume $\omega \ll \omega_{max}$.

(c) [2] In Fig 1-3, Alice uses a motor positioned at a height h above the ground to set the cylinder spinning with an initial angular velocity ω_0. In a prank, Bob uses his "psychic powers" to release the cylinder from the motor without altering its angular velocity. The cylinder, starting with a negligible initial translational velocity, falls toward the ground, which has a coefficient of friction μ. Ignoring air resistance, assume that collisions with the ground are perfectly elastic in the y-direction and that the collision duration t_c is negligible ($gt_c^2 \ll h$), where g represents the acceleration due to gravity. Sketch a qualitative graph representing the trajectory of the cylinder's center of mass.

(d) The cylinder hits the ground at O at $t = 0$.

(di) [2] Find the velocity (v_x, v_y) and the angular velocity ω of the cylinder right after the 1$^{\text{st}}$ collision.

(dii) [3] Find the position of the ball $x(t)$ and $y(t)$, as well as the angular velocity $\omega(t)$. Express your answers in terms of the number of collisions with the ground N (We take $N = 1$ at $t = 0$), the time elapsed since the last collision $\Delta t, h, g, r$, and the ratio $K = \frac{mr^2}{I}$. You can use the floor function $\lfloor x \rfloor$ which denotes the integer part of x (e.g. $\lfloor \pi \rfloor = 3$, $\lfloor e \rfloor = 2$).